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CHAPTER 1

COMPLEX NUMBERS

The equation $x^2 + 1 = 0$, has no solution $x \in R$. To solve this equation, we have to introduce extranumbers into our number system. To do this, we define the number i by $i^2+1 = 0$, and then extend the field of all real numbers by adjoining the number i, which is then combined with the real numbers by theoperations addition and multiplication in accordance with the Field axioms of the real number system.

The numbers a + ib, where a; b $\in$R, of the extended field are then added and multiplied in accordancewith the field axioms, suitably extended, and the restriction $i^2 + 1 = 0$.

Imaginary Unit Number

It is defined as the number whose square is (- 1) , and it will be denoted by the letter (i) , i.e.

$$i = \sqrt{-1}\,, \quad i^2 = -1\,, \quad i^3 = -i\,, \quad i^4 = +1\,, \ldots\ldots$$

i.e. $i^{2n} = (-1)^n$; $i^{2n+1} = (-1)^n i$

where n is an integer. Hence, we may write

$$\pm\sqrt{-4} = \pm\sqrt{4}\,i = \pm 2i$$

$$\frac{1}{2}\left(-1 \pm \sqrt{-3}\right) = \frac{1}{2}\left(-1 \pm \sqrt{3}i\right)$$

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COMPLEX NUMBER

Suppose that $z = x + iy$, where x; y ∈ R. The real number x is called the real part of z, and denotedby x = Re (z).

The real number y is called the imaginary part of z, and denoted by y = Im (z). The set $C = \{z : z = x + iy\ ; x, y \in R\}$ is called the set of all complex numbers. The complex number $\overline{z} = x - iy$,iscalled the conjugate of z.It is easy to see that for every z ∈C, we have

(*i*) $|z| = |\overline{z}|$

(*ii*) $\arg(\overline{z}) = -\arg(z)$

(*iii*) $z + \overline{z} = 2x = 2x = 2\,\mathrm{Re}(z)$

(*iv*) $z - \overline{z} = 2iy = 2\,\mathrm{Im}(z)$

$$x = \mathrm{Re}(z) = \frac{z + \overline{z}}{2} \quad ; \quad y = \mathrm{Im}(z) = \frac{z - \overline{z}}{2i}$$

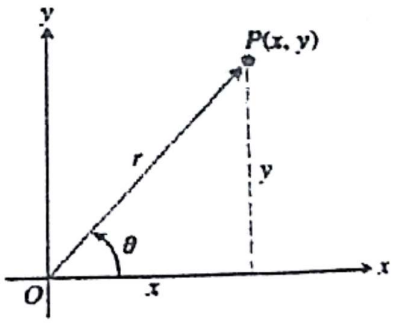
Furthermore, if w ∈C, then

$\overline{z + w} = z + \overline{w}$; $\overline{z\,w} = \overline{z}\ \overline{w}$

ARGAND PLANE

There are two geometric representations of the complex number

1. as the point $P(x, y)$ in the xy-plane
2. as the vector from the origin to P.



In each representation, the x-axis is called the **real axis** and the y-axis is the **imaginaryaxis**. Both representations are **Argand diagrams** for $z = x + iy$.

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In terms of the polar coordinates of x and y, we have

$$x = r\cos\theta \quad , \quad y = r\sin\theta$$

$$z = x + iy = r\cos\theta + ir\sin\theta$$

$$z = r(\cos\theta + i\sin\theta) \quad (1)$$

We define the **absolute value** of a complex number $z = x + i\,y$ to be the length r of a vector $\overline{OP}$ from the origin to $P(x, y)$. We denote the absolute value by vertical bars; thus,

$$r = |x + i\,y| = \sqrt{x^2 + y^2}$$

Note, however, that for a given $z \in C$, argz is not unique. Clearly we can add any integer multiple of$2\pi$to θwithout affecting (1).

The complex number z can be written as:

$$z = r[\cos(\theta + 2\pi k) + i\sin(\theta + 2\pi k)]$$
$$z = r\,cis(\theta + 2\pi k)$$

$$\tan\theta = \frac{y}{x} \quad or \quad \theta = \tan^{-1}\frac{y}{x}$$

$$\cos\theta = \frac{x}{\sqrt{x^2 + y^2}} = \frac{x}{r} \quad ; \quad \sin\theta = \frac{y}{\sqrt{x^2 + y^2}} = \frac{y}{r}$$

Note, however, that for a given z ∈C, argz is not unique. Clearly we can add any integer multiple of$2\pi$to θwithout affecting (1).

The complex number z can be written as:

$z = r[\cos(\theta + 2\pi k) + i\sin(\theta + 2\pi k)]$
$z = r\,cis(\theta + 2\pi k)$
or

$$z = re^{i(\theta + 2\pi k)} \quad , \quad k = 0, 1, 2, 3, \cdots$$

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We sometimes call a real number θ ∈R the principal argument of z if itsatisfies the equations

$$-\pi \le \arg(z) \le \pi$$

The principal argument of z is usually denoted by Argz.

The following equation gives a useful formula connecting a complex number z, itsconjugate $\overline{z}$ and its absolute value namely,

$$|z|^2 = z.\overline{z}$$

EULER'S FORMULA

The identity

$$e^{i\theta} = \cos\theta + i\sin\theta$$

called**Euler's formula**, enables us to rewrite

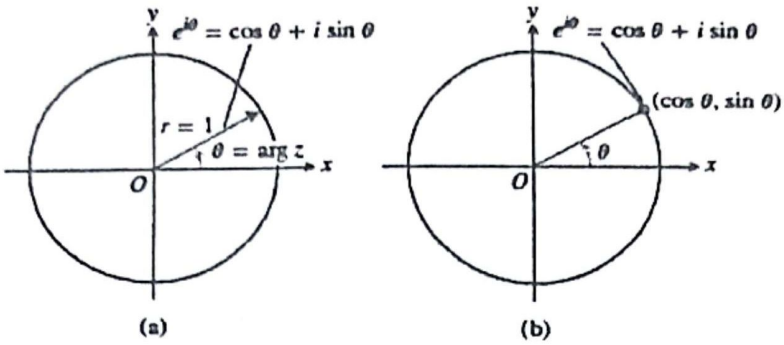
$$z = r(\cos\theta + i\sin\theta)$$

as

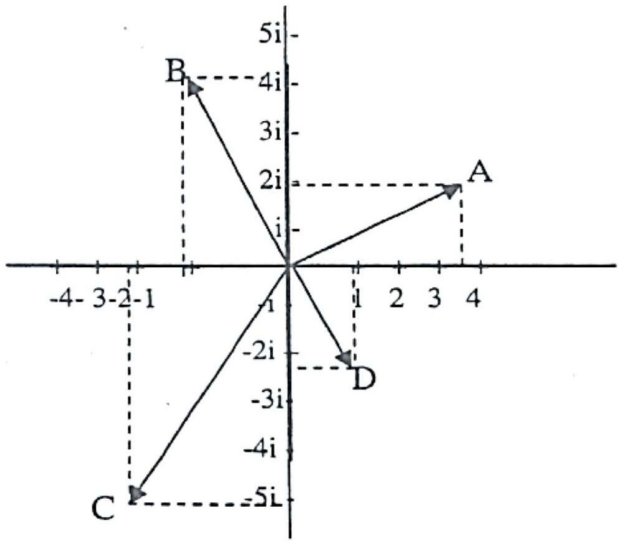
$$z = re^{i\theta} \quad or \quad z = r\,cis\theta$$

This formula, in turn, leads to the following rules for calculating products, quotients, powers,and roots of complex numbers. It also leads to Argand diagrams for $e^{i\theta}$. Since $e^{i\theta} = \cos\theta + i\sin\theta$, is what we get from Equation (1) by taking r = 1, we can say that is represented by a unit vector that makes an angleθ with the + ve x-axis, as shown in the Figure .

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In the following Fig. point A represents the complex number(3+2i) and B represents the complex number (-2+4i). Also, points C and D represented by (-3 -5i) , (1 - 3i)



EXAMPLE 1

Express $z = -\sqrt{3} + i$, in the form of amplitude and argument

SOLUTION

$$Amplitude = r = |z| = \sqrt{x^2 + y^2}$$

$$= \sqrt{3+1} = 2$$

$$\tan\theta = \frac{y}{x} = \frac{1}{-\sqrt{3}}$$

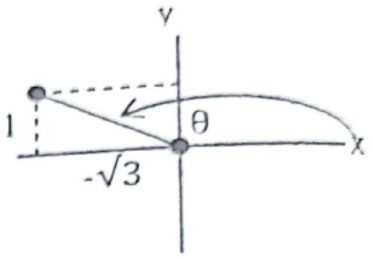
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From the Fig. The number lies in the second quadrant, since

$\cos\theta = \frac{x}{r} = \frac{-\sqrt{3}}{2} \Rightarrow$ 2nd or 3rd quadrant

$\sin\theta = \frac{y}{r} = \frac{1}{2} \Rightarrow$ 1st or 2nd quadrant

$$\therefore \theta = \frac{5\pi}{6}$$



The amplitude and argument form is

$$z = 2\,cis\left(\frac{5\pi}{6}\right) \quad or \quad z = 2e^{i\left(\frac{5\pi}{6}\right)}$$

OPERATIONS ON COMPLEX NUMBERS

1. Addition and subtraction two Complex numbers:

$$z_1 = x_1 + iy_1 \,,\; z_2 = x_2 + iy_2$$

$$z = z_1 + z_2 = (x_1 + x_2) + i(y_1 + y_2)$$

$$z = z_1 - z_2 = (x_1 - x_2) + i(y_1 - y_2)$$

Geometrically, the sum of the two complex numbers z_1, z_2 can be represented by the diameter of the parallelepiped $\overline{OP}$, where z_1, z_2 are two adjacent sides in it.

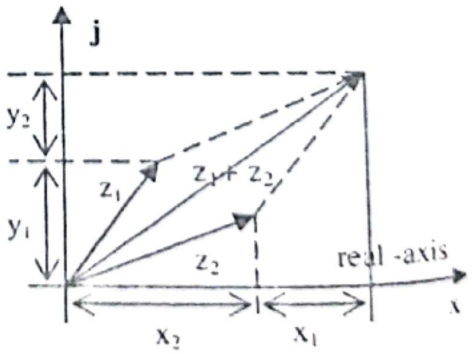


Fig. a

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While the difference $(z_1 - z_2)$ is represented by the other diagonal. Also, $|z_1 - z_2|$ represent the distance between them as in the figure.

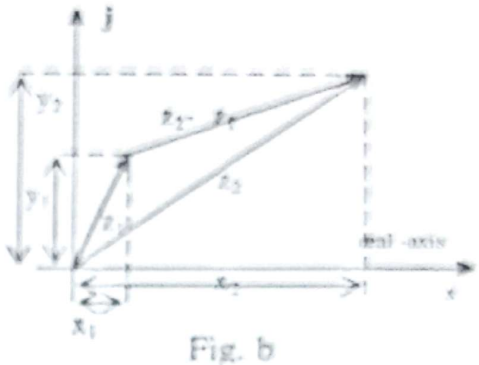


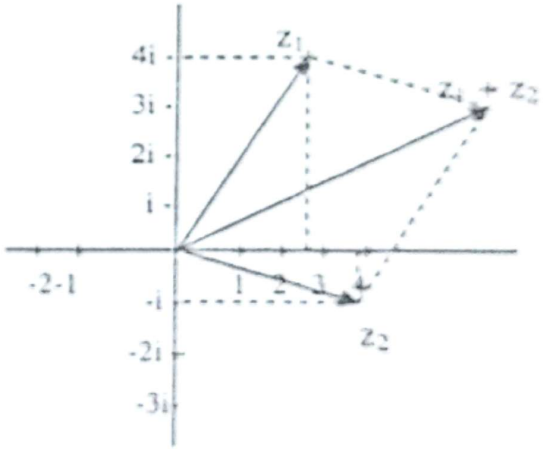
Fig. b

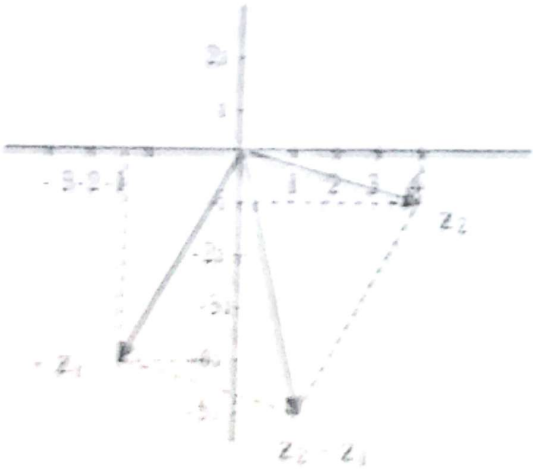
EXAMPLE 2►

If $z_1 = 2 + 4i$, $z_2 = 3 - i$, determine $z_1 + z_2$, $z_2 - z_1$, and draw the results on Argand diagram.

SOLUTION

$$\begin{aligned} z_1 + z_2 &= (2 + 4i) + (3 - i) \\ &= (2 + 3) + (4i - i) \\ &= 5 + 3i \end{aligned}$$





$$\begin{aligned} z_2 - z_1 &= (3 - i) - (2 + 4i) \\ &= (3 - 2) + (-4i - i) \\ &= 1 - 5i \end{aligned}$$

Product Of Complex Numbers

Let

$$z_1 = x_1 + iy_1 = r_1 cis(\theta_1), \quad z_2 = x_2 + iy_2 = r_2 cis(\theta_2)$$

Then

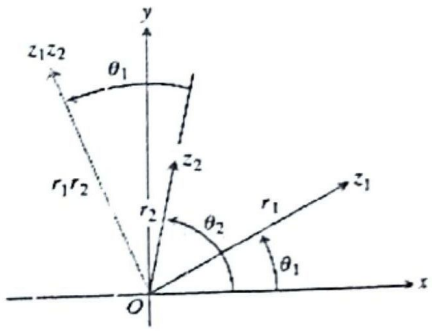
$$z_1 \cdot z_2 = (x_1x_2 - y_1y_2) + i(x_1y_2 - x_2y_1)$$

$$= r_1r_2 cis\theta_1 \cdot cis\theta_2 = r_1e^{i\theta_1} \cdot r_2e^{i\theta_2}$$

$$= r_1r_2e^{i(\theta_1+\theta_2)} = r_1r_2 cis(\theta_1 + \theta_2)$$

$$= r_1r_2[\cos(\theta_1 + \theta_2) + i\sin(\theta_1 + \theta_2)]$$

Thus, the product of two complex numbers is represented by a vector whose length is theproduct of the lengths of the two factors and whose argument is the sum of their arguments as in the Figure.



In particular, from Equation (12) a vector may be rotated counterclockwisethrough an angle θby multiplying it by $e^{i\theta}$. Multiplication by *i* rotates 90°, by(-1) rotates180°,by (- i) rotates 270°, and so on.

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EXAMPLE3

Let $z_1 = 1+i$, $z_2 = \sqrt{3} - i$. Calculate $z_1 . z_2$.

SOLUTION

At first we transform the numbers into polar form

$$z_1 = 1+i \Rightarrow |z_1| = \sqrt{1+1} = \sqrt{2}$$

$$\tan\theta_1 = \frac{1}{1} = 1 \Rightarrow \theta_1 = \frac{\pi}{4}$$

$$z_1 = \sqrt{2}\, e^{\pi/4}$$

$$z_2 = \sqrt{3} - i \Rightarrow |z_2| = \sqrt{3+1} = 2$$

$$\sin\theta_2 = \frac{-1}{2} \Rightarrow \theta_2 = 210^0 = -150^0 = -\frac{5\pi}{6} \text{ or } 330^0 = -30^0 = -\frac{\pi}{6}$$

$$\cos\theta_2 = \frac{\sqrt{3}}{2} \Rightarrow \theta_2 = 30^0 = \frac{\pi}{6} \text{ or } 330^0 = -30^0 = -\frac{\pi}{6}$$

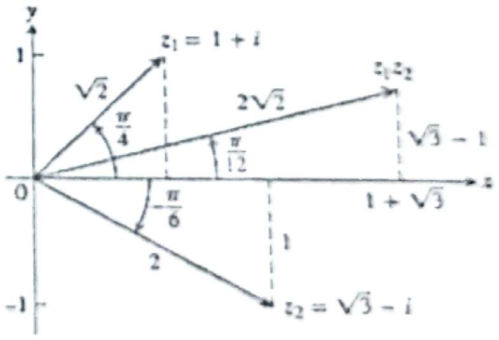
Hence, $\theta_2 = -\frac{\pi}{6}$

$$z_2 = 2\, e^{-\pi/6}$$

$$z_1 . z_2 = \sqrt{2}\, e^{\pi/4} \times 2\, e^{-\pi/6}$$

$$= 2\sqrt{2}\, e^{\pi/4 - \pi/6}$$

$$= 2\sqrt{2}\, e^{\pi/12}$$



$$z_1 . z_2 = 2\sqrt{2}\left[\cos(\pi/12) + i\sin(\pi/12)\right]$$

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DIVISION OF COMPLEX NUMBERS

If $z_2 \neq 0$, then

$$\frac{z_1}{z_2} = \frac{x_1 + iy_1}{x_2 + iy_2}$$

Multiply and divide by the complex conjugate $\overline{z}_2$

$$\frac{z_1}{z_2} = \frac{x_1 + iy_1}{x_2 + iy_2} \cdot \frac{x_2 - iy_2}{x_2 - iy_2}$$
$$= \frac{(x_1x_2 + y_1y_2) + (x_2y_1 - x_1y_2)}{(x_2^2 + y_2^2)}$$

In the polar form, we may write

$$z_1 = r_1 e^{i\theta_1} = r_1 cis(\theta_1)$$
$$z_2 = r_2 e^{i\theta_2} = r_2 cis(\theta_2)$$

$$\therefore \frac{z_1}{z_2} = \frac{r_1 e^{i\theta_1}}{r_2 e^{i\theta_2}} = \frac{r_1}{r_2} e^{i(\theta_1 - \theta_2)} = \frac{r_1}{r_2} cis(\theta_1 - \theta_2).$$
$$= \frac{r_1}{r_2}[\cos(\theta_1 - \theta_2) + i\sin(\theta_1 - \theta_2)]$$

That is, we divide lengths and subtract angles for the quotient of complex numbers.

Remarks:

(1) $$\frac{1}{z} = \frac{1}{r} \cdot \frac{1}{\cos\theta + i\sin\theta}$$

$$= \frac{1}{re^{i\theta}} = \frac{1}{r} e^{-i\theta} = \frac{1}{r} cis(-\theta)$$

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2) $1 = cis(\theta) = cis(0) = \cos 0 + i \sin 0$

$-1 = cis(\pi) = \cos \pi + i \sin \pi$

$i = cis\left(\frac{\pi}{2}\right) = \cos \frac{\pi}{2} + i \sin \frac{\pi}{2}$

$-i = cis\left(-\frac{\pi}{2}\right) = \cos\left(-\frac{\pi}{2}\right) + i \sin\left(-\frac{\pi}{2}\right)$

EXAMPLE 4

Let $z_1 = 1 - 3i$, $z_2 = -2 + 5i$, $z_3 = -3 - 4i$, Calculate the following numbers

(a) $z_1 z_2$, (b) $\frac{z_1}{z_3}$

(c) $\frac{z_1 z_2}{z_1 + z_2}$, (d) $z_1 z_2 z_3$

SOLUTION

(a) $z_1 z_2 = (1 - 3i)(-2 + 5i)$

$= -2 + 5i + 6i - 15i^2$

$= -2 + 11i + 15 = 13 + 11i$

(b) $\frac{z_1}{z_3} = \frac{1 - 3i}{-3 - 4i} = \frac{1 - 3i}{-3 - 4i} \times \frac{-3 + 4i}{-3 + 4i}$

$= \frac{(1 - 3i) \times (-3 + 4i)}{3^2 + 4^2}$

$= \frac{-3 + 4i + 9i - 12i^2}{3^2 + 4^2} = \frac{-3 + 13i + 12}{25}$

$= \frac{9 + 13i}{25} = \frac{9}{25} + i\frac{13}{25}$

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$$(c)\ \frac{z_1 z_2}{z_1 + z_2} = \frac{13+11i}{(1-3i)+(-2+5i)}$$

$$= \frac{13+11i}{-1+2i} = \frac{13+11i}{-1+2i} \times \frac{-1-2i}{-1-2i}$$

$$= \frac{-13-11i-26i+22}{1+4}$$

$$= \frac{9-37i}{5} = \frac{9}{5} - i\frac{37}{5}$$

$$(d)\ z_1 z_2 z_3 = (13+11i)\times(-3-4i)$$

$$z_1 z_2 z_3 = 5 - 85i$$

Note:

We can use the formulas

$$x = \frac{z+\bar{z}}{2} \quad and \quad y = \frac{z-\bar{z}}{2j} \quad (1)$$

toexpress f in terms of the variables z and $\bar{z}$.

EXAMPLE 5►

Express f(z) = 4 x^2 + j (4y^2) by a formula involving the variables and $\bar{z}$.

SOLUTION. Calculation reveals that

$$f(z) = \left(\frac{z+\bar{z}}{2}\right)^2 + 4j\left(\frac{z-\bar{z}}{2j}\right)^2$$

$$= z^2 + 2z\bar{z} + \bar{z}^2 - j(z^2 - 2z\bar{z} + \bar{z}^2)$$

$$= (1-j)z^2 + (2+2j)z\bar{z} + (1-j)\bar{z}^2 \ \#$$

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Special Case

If n is a positive integer, we may apply the product formulas

$$z_1 \cdot z_2 = r_1 r_2 e^{i(\theta_1 + \theta_2)}$$

$$= r_1 r_2 [\cos(\theta_1 + \theta_2) + i \sin(\theta_1 + \theta_2)]$$

to obtain

$$z^n = r^n(\cos n\theta + i \sin n\theta) \;\; ; \;\; n = 1, 2, 3, \ldots.$$

If we take $r = 1$, we obtain De Moivre's Theorem in the form

$$(\cos\theta + i \sin\theta)^n = (\cos n\theta + i \sin n\theta) \;\; ; \;\; n = 1, 2, 3, \ldots.$$

Hence, we may write

$$z^n = [r cis(\theta)]^n = r^n cis(n\theta)$$

If n is a negative number, then

$$z^{-n} = r^{-n} cis(-n\theta)$$

$$\frac{1}{z^n} = \frac{1}{r^n}[\cos\theta - i \sin\theta]$$

If n is a ± vefraction , $n = p/q$, where p, q are integers, then

$$(cis\theta)^{p/q} = (cis\, p\theta)^{1/q}$$

Suppose we put

$$(cis\, p\theta)^{1/q} = cis\varphi \;\; (1)$$

$$\therefore cis(p\theta) = cis(q\varphi)$$

This leads to

$$q\varphi = p\theta + 2\pi k \quad \Rightarrow \quad \varphi = \frac{p\theta + 2\pi k}{q} \;\; (2)$$

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$${}^4C_1 = 4,\ {}^4C_2 = \frac{4\times 3}{2\times 1} = 6,\ {}^4C_3 = 4,\ {}^4C_4 = 1$$

$$\cos^4\theta = \frac{1}{16}\left[e^{4i\theta} + 4e^{2i\theta} + 6 + 4e^{-2i\theta} + e^{-4i\theta}\right]$$

$$= \frac{1}{16}\left[6 + 4\left(e^{2i\theta} + e^{-2i\theta}\right) + \left(e^{-4i\theta} + e^{4i\theta}\right)\right]$$

$$= \frac{1}{16}[6 + 8\cos 2\theta + 2\cos 4\theta]$$

$$\cos^4\theta = \frac{1}{8}\cos 4\theta + \frac{1}{2}\cos 2\theta + \frac{3}{8}$$

$$b)\ \sin^3\theta = \left(\frac{e^{i\theta} - e^{-i\theta}}{2i}\right)^3 = \frac{\left(e^{i\theta} - e^{-i\theta}\right)^3}{-8i}$$

$$= \frac{i}{8}\left[e^{3i\theta} - 3e^{i\theta} + 3e^{-i\theta} - e^{-3i\theta}\right]$$

$$= \frac{i}{8}\left[\left(e^{3i\theta} - e^{-3i\theta}\right) - 3\left(e^{i\theta} - e^{i\theta}\right)\right]$$

$$= \frac{i}{8}[2i\sin 3\theta - 3\times 2i\sin\theta]$$

$$\sin^3\theta = -\frac{1}{4}\sin 3\theta + \frac{3}{4}\sin\theta$$

ROOTS OF COMPLEX NUMBERS

Any complex number w is called the n^{th} root of the complex nu z, if $w^n = z$, and is expressed as: $w = z^{1/n}$

From De Moivre's theorem

$$z^{1/n} = [r(\cos\theta + i\sin\theta)]^{1/n}$$

$$z^{1/n} = r^{1/n}\left\{\cos\frac{\theta + 2k\pi}{n} + i\sin\frac{\theta + 2k\pi}{n}\right\},\ k = 0, 1, 2, \cdots n-$$

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From which there will be n – different values for $z^{1/n}$, i.e. there will be n roots for the complex number $z^{1/n}$.

The n^{th} –roots of unity

The solutions of the equation $z^n = 1$, $n \in I$, are called the nth roots of unity.

$z = 1 \Rightarrow z = \cos 0 + i \sin 0 = 1 \Rightarrow \theta = 0$

the nth roots of unity are thus given by

$$z = \cos\frac{2k\pi}{n} + i\sin\frac{2k\pi}{n}; \quad k = 0, 1, 2, 3, \ldots, n-1$$

If we put

$$w = \cos\frac{2\pi}{n} + i\sin\frac{2\pi}{n} = e^{i2\pi/n}$$

The roots will be

$$1, w, w^2, w^3, \ldots\ldots$$

EXAMPLE9►

Determine all values of z such that $z^5 = -32$

SOLUTION

$$z = (-32)^{1/5} = [32(-1)]^{1/5} = [32(\cos\pi + i\sin\pi)]^{1/5}$$

$$z = 2\left\{\cos\frac{\pi + 2k\pi}{5} + i\sin\frac{\pi + 2k\pi}{5}\right\}, k = 0, 1, 2, 3, 4$$

$$k = 0 \Rightarrow z_1 = 2\left[\cos\frac{\pi}{5} + i\sin\frac{\pi}{5}\right] = 2[\cos 63^0 + i\sin 63^0]$$

$$k = 1 \Rightarrow z_2 = 2\left[\cos\frac{3\pi}{5} + i\sin\frac{3\pi}{5}\right]$$

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$$k=2 \Rightarrow z_3 = 2\left[\cos\frac{5\pi}{5} + i\sin\frac{5\pi}{5}\right] = -2$$

$$k=3 \Rightarrow z_4 = 2\left[\cos\frac{7\pi}{5} + i\sin\frac{7\pi}{5}\right]$$

$$k=4 \Rightarrow z_5 = 2\left[\cos\frac{9\pi}{5} + i\sin\frac{9\pi}{5}\right]$$

EXAMPLE10►

Determine the square roots of the complex number(5 + 12 i) in both polar and Cartesian forms

SOLUTION

Polar form

$$z = 5 + 12i$$

$$\text{mod}(z) = r = \sqrt{(5)^2 + (12)^2} = 13$$

$$Arg(z) = \theta = \tan^{-1}\frac{12}{5} = 67.38^0$$

Square Roots

$$\sqrt{z} = r^{1/2}cis\left(\frac{1}{2}\times 67.38^0\right) \text{ and } \sqrt{z} = r^{1/2}cis\left(\frac{1}{2}\times\left(67.38^0 + 360^0\right)\right)$$

$$\sqrt{z} = \sqrt{13}\ cis\left(\frac{1}{2}\times 67.38^0\right) \text{ and } \sqrt{z} = \sqrt{13}\ cis\left(\frac{1}{2}\times\left(427.38^0\right)\right)$$

$$\sqrt{z} = 3.61\ cis\left(33.69^0\right) \text{ and } \sqrt{z} = 3.61\ cis\left(213.69^0\right)$$

$$\sqrt{z} = 3.61\ cis\left(33.69^0\right) \text{ and } \sqrt{z} = 3.61\ cis\left(-146.31^0\right)$$

Cartesian form

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$$Root\,(1) = 3.61\ cis\left(33.69^0\right)$$
$$= 3.61\cos(33.69) + i\,3.61\sin(33.69)$$
$$= 3 + 2i$$

$$Root\,(2) = 3.61\ cis\left(-146.31^0\right)$$
$$= 3.61\cos(-146.31) + i\,3.61\sin(-146.31)$$
$$= -3 - 2i$$

EXAMPLE 11►

Determine the following roots

$a)\ (-1+i)^{1/3}$, $b)\ \left(-2\sqrt{3}-2i\right)^{1/4}$

SOLUTION

Put $w = -1 + i \Rightarrow z^3 = w$

In the polar form, we have

$$r = \sqrt{1+1} = \sqrt{2}$$

$$\cos\theta = \frac{-1}{\sqrt{2}}, \quad \sin\theta = \frac{1}{\sqrt{2}} \quad \Rightarrow \quad \therefore \theta = 135^\circ = \frac{3\pi}{4}$$

$$w = \sqrt{z}\left(\cos\frac{3\pi}{4} + i\sin\frac{3\pi}{4}\right)$$

$$\therefore z = w^{1/3} = \left(\sqrt{2}\right)^{1/3}\left\{\cos\left(\frac{3\pi/4 + 2k\pi}{3}\right) + i\sin\left(\frac{3\pi/4 + 2k\pi}{3}\right)\right\}$$
$$; k = 0, 1, 2$$

$$k = 0 \Rightarrow z_1 = (2)^{1/6}\left[\cos\frac{\pi}{4} + i\sin\frac{\pi}{4}\right]$$

$$k = 1 \Rightarrow z_2 = (2)^{1/6}\left[\cos\frac{11\pi}{12} + i\sin\frac{11\pi}{12}\right]$$

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$$k=2 \Rightarrow z_3=(2)^{1/6}\left[\cos\frac{19\pi}{12}+i\sin\frac{19\pi}{12}\right]$$

b) Put $w=-2-2i \Rightarrow z^4=w \Rightarrow z=w^{1/4}$

$$r=\sqrt{12+4}=4$$

$$\cos\theta=\frac{-2\sqrt{3}}{4}=-\frac{\sqrt{3}}{2}, \quad \sin\theta=-\frac{2}{4}=-\frac{1}{2}$$

$$\theta=120^\circ=7\pi/6$$

$$z=w^{1/4}=(4)^{1/4}\left\{\cos\frac{7\pi/6+2k\pi}{4}+i\sin\left(\frac{7\pi/6+2k\pi}{4}\right)\right\}$$

The solutions are

$$z_1=\sqrt{2}\left[\cos\left(\frac{7\pi}{24}\right)+i\sin\left(\frac{7\pi}{24}\right)\right]$$

$$z_2=\sqrt{2}\left[\cos\left(\frac{19\pi}{24}\right)+i\sin\left(\frac{19\pi}{24}\right)\right]$$

$$z_3=\sqrt{2}\left[\cos\left(\frac{31\pi}{24}\right)+i\sin\left(\frac{31\pi}{24}\right)\right]$$

$$z_4=\sqrt{2}\left[\cos\left(\frac{43\pi}{24}\right)+i\sin\left(\frac{43\pi}{24}\right)\right]$$

	EXERCISES
1.	Calculate the following complex numbers $(a)\ (2+3i)+(5-i)$ $(b)(-2+6i)-(3-2i)$ $Ans.: 8+i\ ,\ -5+8i$
2.	If $z_1=1+2i\ , z_2=4-3i\ , z_3=-2+3i\ , z_4=-5-i$ Calculate the following complex numbers

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	(a) $z_1 + z_2 - z_3$ (b) $z_2 - z_1 + z_4$ (c) $z_1 z_2 - z_3$ (d) $z_2 + \frac{z_1}{z_4}$ (e) $\frac{z_1 + z_2}{z_3 - z_1}$ (f) $\frac{z_2 z_3}{z_1 + z_4}$
3.	Express the following numbers in polar form and Cartesian form (a) $[1.5\angle 14^0]^5$ (b) $[3\angle 41^0]^4$ (c) $(1+2i)^6$ (d) $(-2-i)^5$ *Ans.*: $7.594\angle 75^0$, $81\angle 164^0$, $125\angle 20.61^0$, $55.9\angle -47.18^0$
4.	Express the following numbers in the polar form (a) $(6+5i)^3$ (b) $(3-8i)^5$ (c) $(-2+7i)^4$ (d) $(-16-9i)^6$
5.	Calculate all values of $(1+i)^{1-i}$
6.	Determine the square roots of the following complex numbers (a) $1+i$ (b) $3-4i$ (c) $7\angle 60^0$ (d) $12\angle \frac{3\pi}{2}$

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7.	Solve the following equations for the real numbers, x and y. $(a)\ 2-3i=\sqrt{a+ib}$ $(b)\ (x-2yi)+(y-3xi)=2+3i$ $(c)(2+i)(3-2i)=a+ib$ $(d)\dfrac{2+i}{1-i}=i(x+iy)$ $(e)\ (3+4i)^2-(2x-iy)=x+iy$ $(f)\left(\dfrac{1+i}{1-i}\right)+\dfrac{1}{x+iy}=1+i$ $(g)(x+iy)(3-2i)=2(x-2iy)+2i-1$
8.	Prove that $-\dfrac{25}{2}\left(\dfrac{1+2i}{3+4i}-\dfrac{2-5i}{-i}\right)=57+24i$
9.	Find the four solution of the equation $z^4-2z^2+4=0$
10.	Find the six solutions of the equation $z^6+2z^3+2=0$
11.	Find the cubic roots of the equation $-8i$

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